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DIFFRACTION PROBLEM OF H-POLARIZED ELECTROMAGNETIC WAVES BY THE BOUNDED GRATINGS WITH REFLECTOR

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Problems of modeling the interaction of electromagnetic radiation with pre-fractal structures, among which are the limited diffraction grating with the reflector have been actual to the present. Therefore, their theoretical study is important, both in terms of developing mathematical tools to solve entangled boundary-value problems, and from the point of view of a more accurate approximation to the real physical models. The purpose of this work is to derive the hypersingular integral equation of the diffraction problem on pre-fractal grating consisting of the finite number of infinitely thin perfectly conducting strips located in the plane of the reflector and the construction of discrete mathematical models with the aid of efficient numerical method of discrete singularities. Discrete mathematical model of this diffraction problem have been carried out. Hypersingular integral equations for the boundary-value Neumann problem have been derived. Radiations patterns of the scattered field from depending of order pre-Cantor grating, of wavenumber, of incident angel for electromagnetic field in the case H polarized have been constructed.

KEY WORDS: diffraction problem, h-polarized electromagnetic waves, bounded gratings with reflector.

ЗАДАЧА ДИФРАКЦИИ H-ПОЛЯРИЗОВАННОЙ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ НА ОГРАНИЧЕННЫХ РЕШЕТКАХ С ОТРАЖАТЕЛЕМ

Несвит К.В.

В последнее время стали весьма актуальными задачи моделирования взаимодействия электромагнитного излучения с пре-фрактальными структурами, среди которых значится ограниченная дифракционная решетка с отражателем. Таким образом, теоретическое исследование таких задач важно как с точки зрения разработки математических инструментов для решения проблем сложных краевых задач, так и с точки зрения более точного приближения к реальным физическим моделям. Целью данной работы является получение гиперсингулярного интегральноо уравнения задачи дифракции на пре-фрактальных решетках, состоящих из конечного числа бесконечно тонких идеально проводящих полос, расположенных в плоскости отражателя, а также построения дискретных математических моделей, ориентированных на разработку эффективного численного метода дискретных особенностей. В работе получена дискретная математическая модель задачи дифракции. Получены гиперсингулярные интегральные уравнения для краевой задачи Неймана. Сконструированы радиационные паттерны рассеянного поля от зависимости от пре-Канторовой решетки, волнового числа, угла входящей волны электромагнитного поля в случае h-поляризации.

КЛЮЧЕВЫЕ СЛОВА: задача дифракции, h-поляризованная электромагнитная волна, ограниченные решетки с отражателем.

ЗАДАЧА ДИФРАКЦІЇ H-ПОЛЯРИЗОВАНОЇ ЭЛЕКТРОМАГНІТНОЇ ХВИЛІ НА ОБМЕЖЕНИХ ГРАТАХ З ВІДБИВАЧЕМ

Несвіт К.В.

Останнім часом стали дуже актуальними задачі моделювання взаємодії електромагнітного випромінювання з пре-фрактальними структурами, серед яких значиться обмежені дифракційні грати з відбивачем. Таким чином, теоретичне дослідження таких задач важливо як з точки зору розробки математичних інструментів для вирішення проблем складних крайових задач, так і з точки зору більш точного наближення до реальних фізичних моделей. Метою даної роботи є отримання гіперсінгулярного інтегрального рівняння задачі дифракції на пре-фрактальних гратах, що складаються зі скінченного числа нескінченно тонких ідеально провідних смуг, розташованих в площині відбивача, а також побудови дискретних математичних моделей, орієнтованих на розробку ефективного чисельного методу дискретних особливостей. В роботі отримана дискретна математична модель задачі дифракції. Отримані гіперсінгулярні інтегральні рівняння для крайової задачі Неймана. Сконструйовані радіаційні патерни розсіяного поля в залежності від пре-Канторової решітки, хвильового числа, кута вхідної хвилі електромагнітного поля в разі h-поляризації.

КЛЮЧОВІ СЛОВА: задача дифракції, h-поляризована електромагнітна хвиля, обмежені решітки з відбивачем.

1.Introduction.

Mathematical modeling of the interaction of electromagnetic wave with electrodynamic structures of many papers related to the solution of applied problems. These structures include pre-fractal diffraction gratings with a reflector.

At this time the actual problem of modeling the interaction of electromagnetic radiation with pre-fractal structures, among which are the bounded diffraction grating with reflector [4-9]. Examples of modern devices, which are based on bounded diffraction gratings are compact discs (DVD), antennas for mobile phones, a template for modern 3D displays.

In papers [1,2,4-11] the boundary value problems of boundary integral equations for the stationary wave equations that result from problems in the theory of electromagnetic waves in plane-parallel structures have been investigated. In paper [9] the diffraction problem of a plane monochromatic wave on a pre-fractal periodic grating with reflector have been investigated and created the discrete mathematical model of a system of singular integral equations (SIE), efficient numerical method of discrete singularities (MDS). In papers [10], [11] mathematical models of wave diffraction problem with few gratings and reflector for the case E and H polarized have been carry out based on SIE.

The purpose of this work is to derive the hypersingular integral equation of the diffraction problem on pre-fractal grating consisting of the finite number of infinitely thin perfectly conducting strips located in the plane of the reflector and the construction of discrete mathematical models based on MDS [2,6,7].

2. Formulation of the problem.

In two dimensions the full electromagnetic field is represented as a superposition of two fields: $(E_x,0,0),(0,H_y,H_z)$ is E polarization and $(0,E_y,E_z),(H_x,0,0)$ is H polarization (time dependence is given by the factor $e^{-i\omega t}$). In this case, the complex amplitude of the full field, $E_x=u(y,z)$ or $H_x=u(y,z)$ satisfies the Helmholtz equation [1,2] over the reflector out of tapes:

$$\Delta u(y,z)+k^2u(y,z)=0,\quad k=\frac{\omega}{c},\tag{1}$$

boundary conditions on the strips and the reflector, the Sommerfeld radiation conditions, and the Meixner condition on the edges strips [1,2].

Pre-Cantor sets of segments called segments $L^{(N)}$ obtained by the principle of constructing a Cantor set on the N-th step [3] (see Fig. 1).

We choose a Cartesian coordinate system so that the grating was in the xOy-plane and the Ox-plane was of parallel to the edges of strips (2).

Need to find the full field $u^{(N)}(y,z)$, resulting from the scattering H polarized monochromatic plane wave on considered diffraction structure (see Fig. 2)

$$\Lambda=\left\{(x,y,z)\in\Re^3,y\in L^{(N)},z=0\right\},$$
$$L^{(N)}=\bigcup_{q=1}^{2^N}\left(a_q^N,b_q^N\right),-l<a_1^N<b_1^N<\ldots<a_{2^N}^N<b_{2^N}^N<l.\tag{2}$$

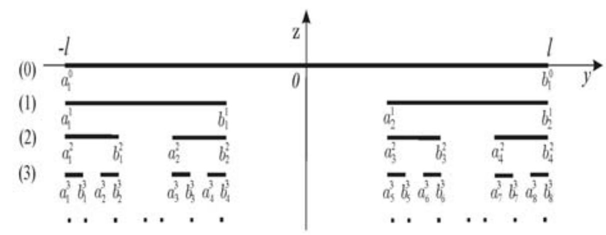

Fig.1. Pre-Cantor sets $L^{(0)},L^{(1)},L^{(2)},L^{(3)}\ldots$

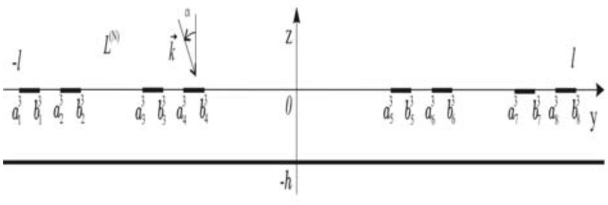

Fig.2. The cross section of the diffraction structure of the plane ***yOz.***

Denote,

$$\Omega_0=\left\{(y,z)\in\Re^2\mid z>0,-l<y<l\right\},\tag{3}$$

$$\Omega_1=\left\{(y,z)\in\Re^2\mid -h<z<0,-l<y<l\right\},\tag{4}$$

where in the field Ω_0, Ω_1 dielectric permittivity is ε_0.

Let the falls H polarized plane electromagnetic wave of unit amplitude from infinity to the top of a diffraction structure at an angle α:

$$u_{inc}^{(N)}(y,z)=H_x(y,z)=e^{ik(y\sin\alpha-z\cos\alpha)}.\tag{5}$$

The full field $u^{(N)}(y,z)$ resulting from diffraction of waves on the observed diffraction structure (see Fig. 2) found in the form

$$u^{(N)}(y,z)=\begin{cases}u_0^{(N)}(y,z)+u_+^{(N)}(y,z), & (y,z)\in\Omega_0,\\ u_0^{(N)}(y,z)+u_-^{(N)}(y,z), & (y,z)\in\Omega_1.\end{cases}\tag{6}$$

where $u_0^{(N)}(y,z)$ is known solution of the Helmholtz equation, represents the sum of incident and reflected waves from the shield (no strips), and $u_+^{(N)}(y,z)$, $u_-^{(N)}(y,z)$ to be determined.

The full field $u^{(N)}(y,z)$ must satisfy the following conditions:

The Helmholtz equation (1) above the shield without the strips

- Boundary conditions on the strips

$$\frac{\partial u^{(N)}(y,z)}{\partial z}\Big|_{z=0}=0,\quad y\in L^{(N)},\tag{7}$$

- and on the shield

$$\frac{\partial u^{(N)}(y,z)}{\partial z}\Big|_{z=-h}=0,\quad y\in\Re, \tag{8}$$

- The conditions of conjugation in the slits

$$u^{(N)}(y,+0)=u^{(N)}(y,-0),\quad y\in CL^{(N)}=[-1,1]\setminus L^{(N)}, \tag{9}$$

$$\frac{\partial u^{(N)}}{\partial z}(y,+0)=\frac{\partial u^{(N)}}{\partial z}(y,-0),\quad y\in CL^{(N)}, \tag{10}$$

- Sommerfeld radiation condition at infinity;
- The condition of Meixner on the edges of strips.

The field $u_0^{(N)}(y,z)$ has the form

$$u_0^{(N)}(y,z)=2\cos\left(k\left(z+h\right)\cdot\cos\alpha\right)\cdot e^{iky\sin\alpha},\quad z>-h,\ y\in\Re. \tag{11}$$

The field $u_+^{(N)}$ in Ω_0, and $u_-^{(N)}$ in Ω_1 found in the form of Fourier represented:

$$u_+^{(N)}(y,z)=-\int_{-\infty}^{\infty}C^{(N)}(\lambda)e^{i\lambda y-\gamma(\lambda)z}d\lambda,\quad (y,z)\in\Omega_0, \tag{12}$$

$$u_-^{(N)}(y,z)=\int_{-\infty}^{\infty}C^{(N)}(\lambda)\frac{ch\left(\gamma(\lambda)(z+h)\right)}{sh\left(\gamma(\lambda)h\right)}e^{i\lambda y}d\lambda,\quad (y,z)\in\Omega_1, \tag{13}$$

where $\gamma(\lambda)=\sqrt{\lambda^2-k^2},\ \mathrm{Re}\,\gamma(\lambda)\ge 0,\ \mathrm{Im}\,\gamma(\lambda)\le 0.$

This choice radical branch $\gamma(\lambda)$ ensures that the of the Sommerfeld radiation conditions has been implemented.

3. Development of hypersingular integral equations (HSIE).

The boundary conditions (7) - (8) and the conditions at the slots (9) - (10), we obtain the dual integral equation:

$$\int_{-\infty}^{\infty}C^{(N)}(\lambda)\left(1+cth\left(\gamma(\lambda)h\right)\right)\cdot e^{i\lambda y}d\lambda=0,\quad y\in CL^{(N)}, \tag{14}$$

$$\int_{-\infty}^{\infty}\gamma(\lambda)C^{(N)}(\lambda)e^{i\cdot\lambda\cdot y}d\lambda=f^{(N)}(y),\quad y\in L^{(N)}. \tag{15}$$

We introduce a new function $F^{(N)}(y)$:

$$F^{(N)}(y)=\int_{-\infty}^{\infty}C^{(N)}(\lambda)\left(1+cth\left(\gamma(\lambda)h\right)\right)\cdot e^{i\cdot\lambda\cdot y}d\lambda=0,\quad y\in\Re. \tag{16}$$

The inverse Fourier transform of function (16)

$$C^{(N)}(\lambda)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{F^{(N)}(\xi)e^{-i\lambda\xi}}{\left(1+cth\left(\gamma(\lambda)h\right)\right)}d\xi. \tag{17}$$

Parametric representation of the hypersingular integral operator [2,5,8]:

$$G(y)=\int_{-\infty}^{\infty}A(\lambda)e^{i\lambda y}d\lambda,$$
$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{G(y)}{(y-\xi)^2}dy=-\int_{-\infty}^{\infty}|\lambda|A(\lambda)e^{i\lambda\xi}d\lambda. \tag{18}$$

Hypersingular integral equation on system of intervals have been created on the base paper [4,5,8] of (14), (15) considering (16)-(19)

$$\frac{1}{\pi}\int_{L^{(N)}}\frac{F^{(N)}(\xi)d\xi}{(\xi-y)^2}+\frac{A}{\pi}\int_{L^{(N)}}\ln|\xi-y|F^{(N)}(\xi)d\xi+\frac{1}{\pi}\int_{L^{(N)}}K^{(N)}(\xi,y)F^{(N)}(\xi)d\xi=f^{(N)}(y),\quad y\in L^{(N)}. \tag{19}$$

where

$$A=-\frac{k^2}{2},\ K^{(N)}(\xi,y)=\frac{ik^2}{4}\left[\frac{2i}{\pi}\ln k+H_0^{(1)}\left(k|\xi-y|\right)-\frac{2i}{\pi}\ln\left(k|\xi-y|\right)\right]-\frac{k^4}{4}\int_{-\infty}^{+\infty}\frac{e^{i\lambda(y-\xi)}}{\gamma(\lambda)\left(|\lambda|+\gamma(\lambda)\right)^2}d\lambda-$$
$$-\int_{-\infty}^{+\infty}\left[\frac{\gamma(\lambda)}{1+cth\left(\gamma(\lambda)h\right)}-\frac{1}{2}\left(|\lambda|-\frac{k^2}{2\gamma(\lambda)}+\frac{k^4}{2\gamma(\lambda)\left(|\lambda|+\gamma(\lambda)\right)^2}\right)\right]e^{i\lambda(y-\xi)}d\lambda.$$

4. HSIE on standard interval (-1,1).

Introduce the restrictions of functions $F^{(N)}(\xi),f^{(N)}(y)$ on intervals $L_q^{(N)}=\left(a_q^N,b_q^N\right),\quad q=\overline{1,2^N},$

$$F^{(N)}(\xi)|_{\xi\in L_q^{(N)}}=F_q^{(N)}(\xi),\ y\in L_p^{(N)},\ \xi\in L_q^{(N)},\quad q=\overline{1,2^N},$$
$$f^{(N)}(y)|_{y\in L_p^{(N)}}=f_p^{(N)}(y),\ y\in L_p^{(N)},\ \xi\in L_q^{(N)},\quad q=\overline{1,2^N}.$$

Meixner condition will be satisfied if $F_q^{(N)}(\xi)$ represented as:

$$F_q^{(N)}(\xi)=u_q^{(N)}(\xi)\sqrt{\left(b_q^N-\xi\right)\left(\xi-a_q^N\right)},\quad q=\overline{1,2^N}, \tag{20}$$

where the function $u_q^{(N)}(\xi)$ is continuous by Holder.

Thus, we get the system HSIE on system intervals:

$$\frac{1}{\pi}\int_{a_p^N}^{b_p^N}\frac{u_p^{(N)}(\xi)\sqrt{\left(b_p^N-\xi\right)\left(\xi-\alpha_p^N\right)}}{(\xi-y)^2}d\xi+\frac{A}{\pi}\int_{a_p^N}^{b_p^N}u_p^{(N)}(\xi)\sqrt{\left(b_p^N-\xi\right)\left(\xi-a_p^N\right)}\ln|\xi-y|d\xi+$$
$$+\frac{1}{\pi}\sum_{q=1}^{2^N}\int_{a_q^N}^{b_q^N}u_q^{(N)}(\xi)\sqrt{\left(b_q^N-\xi\right)\left(\xi-a_q^N\right)}K_{pq}(y,\xi)d\xi=f_p^N(y),\ y\in L_p^{(N)},\ p=\overline{1,2^N}. \tag{21}$$

Chosen standard interval (-1,1) and display it into intervals (a_q^N, b_q^N), $q=1,\dots,2^N$,

$$g_q^{(N)}:(-1,1)\mapsto\left(a_q^N,b_q^N\right):$$

$$t\mapsto g_q^{(N)}(t)=\frac{b_q^N-a_q^N}{2}t+\frac{b_q^N+a_q^N}{2},\quad -1<t<1,$$

then

$$F_q^{(N)}(\xi)=F_q^{(N)}\left(g_q^{(N)}(t)\right)=$$
$$=v_q^{(N)}(t)\frac{b_q^N-a_q^N}{2}\cdot\sqrt{1-t^2},\quad -1<t<1, \tag{22}$$
$$v_q^{(N)}(t)=u_q^{(N)}\left(g_q^{(N)}(t)\right),$$
$$w_p^{(N)}(t_0)=f_p^{(N)}\left(g_p^{(N)}(t_0)\right),$$
$$M_{qp}^{(N)}(t,t_0)=K_{qp}^{(N)}\left(g_q^{(N)}(t),g_p^{(N)}(t_0)\right).$$

Finally, we obtain system HSIE on standard interval:

$$\frac{1}{\pi}\int_{-1}^{1}\frac{v_q^{(N)}(t)\sqrt{1-t^2}}{(t-t_0)^2}dt+\frac{A}{\pi}\int_{-1}^{1}v_q^{(N)}(t)\sqrt{1-t^2}\ln|t-t_0|dt+$$
$$+\frac{1}{\pi}\sum_{q=1}^{2^N}\int_{-1}^{1}v_q^{(N)}(t)\sqrt{1-t^2}M_{pq}^{(N)}(t_0,t)dt=w_p^N(t_0),\quad |t_0|<1. \tag{23}$$

5. Discrete mathematical model of HSIE.

The discrete mathematical model of integral equations have been created on base mathematical model (23). Discretization boundary equations (23) with the aid of effective numerical method discrete singularities have been carried out. Interpolate the required functions $v_p^{(N)}(t)$ by Lagrange polynomials $v_{p,(n-2)}^{(N)}(t)$, $p=1,\dots 2^N$, the nodes, which coincides with the zeros of the Chebyshev polynomials of the second kind. Thus, we get a system for approximate solutions. Next, using the Gauss quadrature finally obtain a system of linear algebraic equations (SLAE) to determine the approximate value of $v_p^{(N)}(t)$:

$$\sum_{\substack{j=1\\ j\neq k}}^{n-1}v_{q,(n-2)}^{(N)}\left(t_{0j}^n\right)\frac{\left(1-(-1)^{j+k}\right)\left(1-\left(t_{0j}^n\right)^2\right)}{\left(t_{0k}^n-t_{0j}^n\right)^2}\frac{1}{n}-$$
$$-v_{q,(n-2)}^{(N)}\left(t_{0k}^n\right)\frac{n}{2}-\frac{A}{n}\sum_{j=1}^{n-1}v_{q,(n-2)}^{(N)}\left(t_{oj}^n\right)\left(1-\left(t_{0j}^n\right)^2\right)\times$$
$$\times\left[\ln 2+2\sum_{k=1}^{n-1}\frac{T_k\left(t_{0j}^n\right)}{k}T_k\left(t_{ok}^n\right)+\frac{(-1)^j}{n}T_k\left(t_{0k}^n\right)\right]+ \tag{24}$$
$$+\frac{1}{n}\sum_{q=1}^{2^N}\sum_{\substack{j=1,\\ j\neq k}}^{n-1}v_{q,(n-2)}^{(N)}\left(t_{0j}^n\right)M_{pq}^{(N)}\left(t_{0k}^n,t_{0j}^n\right)\left(1-\left(t_{0j}^n\right)^2\right)=$$
$$=w_{p,(n-2)}^{(N)}\left(t_{0k}^n\right),\ p=\overline{1,2^N},\ k=\overline{1,n-1},$$

where $v_{p,(n-2)}^{(N)}(t)$, is Lagrange polynomials of degree $(n-2)$ interpolating the required function $v_p^{(N)}(t)$, где $t_{0j}^n=\cos\frac{j}{n}\pi,\ j=1,\dots,n-1,$ is zero of Chebyshev polynomials second kind. Solve SLAE find the required functions in the collocation points and calculate basic coefficients for finding of the scattered and diffracted fields.

6. The radiation pattern.

Discretization of coefficient $C^{(N)}(\lambda)$ (17) have been carried out by construction pattern of the scattered field

$$C^{(N)}(\lambda)=\frac{1}{\pi\left(1+\mathrm{cth}(\gamma(\lambda)h)\right)}\sum_{q=1}^{2^N}\sum_{k=1}^{n}v_q^{n-1}(g_q(t_k^n))\frac{e^{-i\lambda g_q(t_k^n)}-1}{2i\lambda n}, \tag{25}$$

and using the asymptotic representation at $r\to\infty$:

$$H_0^1(kr)\sim\sqrt{\frac{2}{\pi kr}}e^{i\left(kr-\frac{\pi}{4}\right)},$$
$$u_+^{(N)}(r,\varphi)\sim-\int_{-k}^{k}C^{(N)}(\lambda)e^{ir\left(\lambda\cos\varphi-\sqrt{\lambda^2-k^2}\sin\varphi\right)}d\lambda, \tag{26}$$
$$u_-^{(N)}(r,\varphi)\sim\int_{-k}^{k}C^{(N)}(\lambda)\frac{\mathrm{ch}\left(\sqrt{\lambda^2-k^2}\left((r\cdot\sin\varphi)+h\right)\right)}{\mathrm{sh}\left(\sqrt{\lambda^2-k^2}\cdot h\right)}e^{i\lambda r\cos\varphi}d\lambda,$$

approximate formulas for the radiation pattern of the scattered field in the far zone have been carried out

$$D_\pm(\varphi)=\lim_{r\to\infty}\frac{u_\pm(r,\varphi)}{\sqrt{\frac{2}{\pi kr}}e^{i\left(kr-\frac{\pi}{4}\right)}},\quad r=\sqrt{y^2+z^2}. \tag{27}$$

Using the results of [6], [7], we can estimate the rate of convergence of the approximate solutions to the exact metric introduced in Hilbert spaces, and in the uniform metric for physical quantities.

7. The numerical experiment.

The numerical experiment with the aid of an effective method of discrete singularities [6-8] have been carried out. The results are presented in fig.3-6.

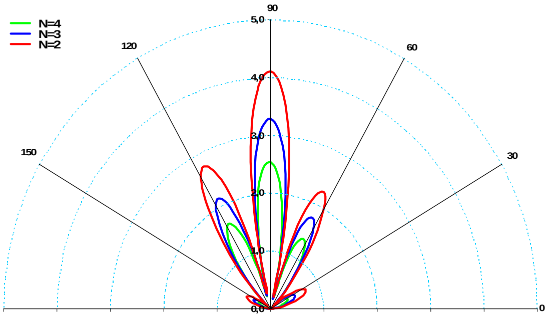

Fig. 3. Radiation patterns from depending N order of pre-Cantor grating by $l=1$, $h=0{,}2$, $k=3\pi$, $\alpha=0^0$.

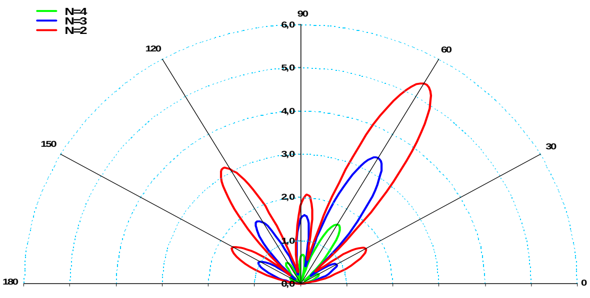

Fig. 4. Radiation patterns from depending N order of pre-Cantor grating by $l=1$, $h=0{,}2$, $k=3\pi$, $\alpha=30^0$.

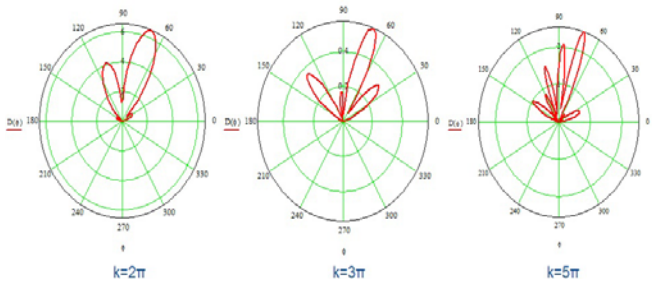

Fig. 5. Radiation patterns of the scattered field over the strips, depending of k from N=3, l=1, h=0,25, α=20⁰. k=2π.

8. Conclusions.

Discrete mathematical model of the diffraction problem of H polarized electromagnetic waves on the gratings consisting of a finite number of perfectly conducting strips as pre-Cantor set with a perfectly conducting reflector with the aid of effective numerical method of discrete singularities (MDS) have been carried out. Hypersingular integral equations for the boundary-value Neumann problem have been derived. Radiations patterns of the scattered field from depending of order pre-Cantor grating, of wavenumber, of incident angel for electromagnetic field in the case H polarized have been constructed.

Future prospects in this field is the investigation of problems of the theory of diffraction by periodic and bounded gratings with impedance boundary conditions, the construction of discrete mathematical models of these problems and carrying out numerical experiments based on them, the numerical solution using the MDS for periodic gratings lying on a dielectric shield.

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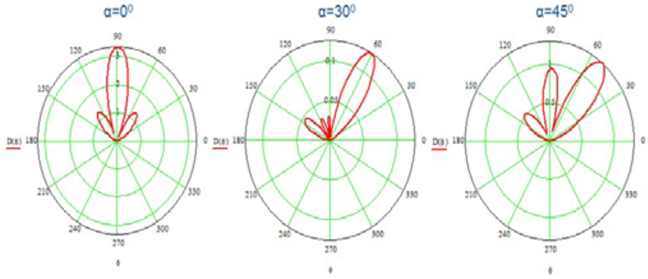

Fig. 6. Radiation patterns of the scattered field over the strips, depending of incidence angle from N=3, l=1, h=0,25,